SPACE-CHARGE EFFECTS ON SYNCHROTRON OSCILLATIONS

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An approximate equation of motion for synchrotron oscillations in the presence of space charge is

$$\frac{d}{dt}(E - E_s) = feV \sin\phi - f\Gamma eV + 2\pi hfeg\,\frac{\partial\lambda}{\partial\phi}\,, \tag{1}$$

$$\frac{d\phi}{dt} = -2\,h\,\frac{\partial f}{\partial E}\,(E - E_s)\,, \tag{2}$$

where f is the revolution frequency, hf the radio frequency, E_s the synchronous energy, ϕ the phase of the rf when the particle crosses the gap, $\Gamma = \sin\phi_s$ the synchronous phase, λ is the space charge per unit length along the circumference, g is the capacitance per unit length of the beam in the vacuum chamber, and V is the total voltage per turn (esu). Equation (1) includes the electrostatic space charge field only. It is shown in reference (1) that the effect of including magnetic interaction is essentially to add a factor γ^{-2} to the third term on the right in Eq. (1). An appropriate formula for g is[1]

$$g = 1 + 2\,\ln\frac{2G}{\pi a}\,, \tag{3}$$

where G is the total vacuum chamber aperture and a is the minor radius of the beam. We will assume that the density λ of the bunch is of the form

$$\lambda = \frac{2eN}{\pi R\Delta}\left[1 - \frac{(\phi - \phi_s)^2}{\Delta^2}\right]^{1/2}\,, \tag{4}$$

which corresponds to h uniformly filled ellipses in phase space with a phase amplitude $\pm\Delta$ and with a total of N particles. We substitute Eq. (4) in Eq. (1) and linearize to obtain

$$\frac{d}{dt}(E - E_s) = \left[feV \cos \phi_s - \frac{4e^2Nghf}{R\Delta^3\gamma^2} \right] (\phi - \phi_s). \tag{5}$$

The phase oscillation frequency given by Eqs. (5) and (2) is

$$\nu_s = \frac{\omega_s}{2\pi f} = \nu_{s0} \left[1 - \frac{4hNg}{\gamma^2\Delta^3 \cos \phi_s} \left(\frac{e}{RV}\right) \right]^{1/2}, \tag{6}$$

where

$$\nu_{s0} = \left[\frac{h}{2\pi} \; \frac{eV}{f} \frac{\partial f}{\partial e} \cos \phi_s \right]^{1/2}. \tag{7}$$

The condition that the phase oscillations are not significantly affected by space charge forces is then

$$\left| \frac{4hNg}{\gamma^2\Delta^3 \cos \phi_s} \left(\frac{e}{RV}\right) \right| = |K| << 1. \tag{8}$$

If the quantity K is of order unity or greater, then space charge forces will be important. The limiting number of particles above which the (small) phase oscillations become unstable is, (if $\cos \phi_s > 0$),

$$N_\ell = \frac{\gamma^2\Delta^3 \cos \phi_s}{4gh} \left(\frac{RV}{e}\right). \tag{9}$$

However, the true limit is presumably obtained by putting $\Delta \approx 1$, since if $K > 1$ for a small value of Δ, the space charge forces will expand the bunch (by weakening the focusing) and increase Δ until $K < 1$. The result (9) then agrees with the result obtained in reference (2) by a more rigorous treatment.

Above the transition energy, $\partial f/\partial E < 0$, and $\cos\phi_s = -\sqrt{1 - \Gamma^2}$. The space charge forces then increase ν_s if $K \gtrsim 1$.

In any case, it is clear that careful attention must be paid to space charge effects if condition (8) is not satisfied.

If the phase oscillations behave adiabatically, the phase spread Δ is given by

$$\Delta^2 = \frac{h}{\nu_s}\frac{1}{f}\frac{\partial f}{\partial E}\Delta_0(E_m - E_s)_0, \tag{10}$$

where $\pm\Delta_0$ and $\pm(E_m - E_s)_0$ are the initial phase and energy spread in the beam. If we neglect space charge, and use Eq. (7), this becomes

$$\Delta^2 = \left[\frac{2\pi h}{eVf\cos\phi_s}\ \frac{\partial f}{\partial e}\right]^{1/2}\Delta_0(E_m - E_s)_0. \tag{11}$$

The change in Δ during acceleration comes mainly from the factor $(\partial f/\partial E)^{1/4}$ which goes to zero at transition where space charge effects will certainly become important.

For the proposed 200 GeV accelerator, we have near injection

	Main Ring	Booster
h	1120	84
g	3	3
R	10^5 cm	7.5×10^3 cm
V	3.47×10^6 V	2.41×10^5 V
γ	11.66	1.213
$\cos\phi_s$	.866	.866
N	5×10^{13}	3.8×10^{12}
Δ	.44	1.38
K	0.028	0.09
N_0	2×10^{16}	1.6×10^{13}

It appears that at injection neither booster nor main ring is in trouble, although a reduction of Δ by a factor of only 2 would make space charge important in the booster. The calculation should be made at other points in the acceleration cycle to be sure that space charge can be ignored everywhere (except at transition).

REFERENCES

1. Nielsen, Sessler, Symon, "Longitudinal Instabilities in Intense Relativistic Beams," CERN Symposium Proceedings, Sept. 1959, p. 239.

2. C. E. Nielsen and A. M. Sessler, "Longitudinal Space Charge Effects in Particle Accelerators," Rev. Sci. Instr. 30, 80 (59).